

1 **Damage Theory Applied to Fatigue Tests in Gap-Graded Asphalt Mixtures** 2 **with 4th Generation Rubberized Asphalt**

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11 **ABSTRACT**

12 Damage to flexible pavements due to stress and strain caused by traffic and
13 environmental factors is manifested in most cases in the form of fatigue cracking. Recent
14 researches have adopted less empirical and more mechanistic criteria for the evaluation of
15 materials used in flexible pavements. Application of the Continuum Damage Theory (CDT) to
16 evaluate the evolution of internal damage in Hot Mix Asphalt (HMA) is investigated in this
17 paper. The CDT leads to the formulation of a characteristic curve relating normalized pseudo-
18 stiffness and the internal damage variable, and this curve is supposed to be unique for each
19 material. New materials, also called 4th generation rubberized asphalt, such as Reacted and
20 Activated Rubber - RAR and Pelletized Asphalt-rubber, were developed with the purpose to
21 simplify the manufacturing process (as in the asphalt mixture obtained by the dry process) and to
22 enhance the performance of the asphalt mixture (as the asphalt mixture obtained by the wet
23 process). The uniqueness of the characteristic curve was investigated in this paper by performing
24 fatigue tests in which prismatic samples of HMA were subjected to cyclic bending loads under
25 strain controlled tests. Asphalt mixtures with gap gradation with 4th generation asphalt-rubber
26 were tested. The results of all the tests showed the existence of unique curves independent of
27 loading mode, amplitude or frequency within the studied range. This property may be
28 implemented in numerical codes to simulate the behaviour of flexible pavements under a variety
29 of field conditions, using a well formulated mechanistic approach.

30 **Keywords:** Fatigue, Damage, Asphalt Rubber.

31 **1. INTRODUCTION**

32 Damage to flexible pavements due to stress and strain caused by traffic and environmental
33 factors is manifested in most cases in the form of fatigue cracking. Fatigue resistance is the
34 ability to withstand repeated loads without cracking and is associated with stiffness of the
35 material [1]. The fatigue characteristics of asphalt are therefore important for the structural
36 design of pavements and they have been investigated using either the phenomenological or
37 mechanistic approach. In the phenomenological approach, the initial strain or stress, and the
38 fatigue life, have been shown to have a power law relationship [2]. Fatigue tests are conducted at
39 various initial stress/strain levels, and the relationship between strain versus number of cycles to
40 failure is the basis for assessing fatigue performance.

1 The dissipated energy [1; 3], the fracture mechanics and the continuum damage mechanics
2 methods may be categorized into a mechanistic approach to study the characteristics of asphalt
3 concrete. Fracture mechanics theories, either linear or non-linear, may be used to track the
4 evolution of existing cracks during numerical analyses. This approach has been trailed by many
5 researchers [4-6] in the pavement community, but it has the intrinsic shortcoming of not being
6 able to forecast the initiation of cracks.

7 The application of the Continuum Damage Theory (CDT) provides an alternative approach
8 for the study of damage initiation and propagation in asphalt mixes. Based on the original works
9 of Schapery (1984, 1990) and Lee (1996), models were developed for Characteristic Curves of
10 hot asphalt mixes subjected to different amplitudes of stress/strain [7-9]. The results showed that
11 these curves were independent of load level, and the evolution of damage was traced by means
12 of an internal damage variable. Other authors reported that the Characteristic Curves are also
13 independent on load frequency and test temperature if the time-temperature superposition
14 principle is applied [10; 11].

15 Cracking can be reduced using modified asphalt binder with the incorporation of ground
16 scrap tyre rubber to produce rubberized asphalt via the wet process [12; 13]. This process results
17 in a highly viscous material with enhanced engineering properties, but requires a specific
18 equipment that is typically installed at the job site, or close to the supplying asphalt plant and its
19 manufacture requires high temperature and long mixing time. New materials such as Reacted and
20 Activated Rubber - RAR and Pellet rubber were developed with the purpose to simplify the
21 manufacturing process (as in the asphalt mixture obtained by the dry process) and to enhance the
22 performance of the asphalt mixture (as the asphalt mixture obtained by the wet process). The
23 RAR, as an asphalt rubber binder, is composed of plain soft bitumen, fine crumb rubber, and an
24 Activated Mineral Binder Stabilizer (AMBS) at optimized proportions. RAR is produced by a
25 short-time hot blending and activation in a specially designed process to form a dried granulated
26 activated rubber [14]. Pellets are produced by mixing crumb rubber and other polymers into the
27 binder and after meeting some specifications, mineral fillers, fibers or other solid components
28 can be added to the pelletized mix [15].

29 The present research analyzed the fatigue resistance of a gap-graded mixtures produced
30 with addition of Pellet and RAR using four point bending tests (4PB) conducted on prismatic
31 samples. These analyzes were carried out according to the work described by Mello et al. (2010)
32 [16] and the tests were performed according to AASHTO T 321-03 [17] and interpreted within
33 the framework of the Continuum Damage Theory.

34 2. THEORETICAL FRAMEWORK

35 Continuum Damage Theory ignores specific micro-scale behaviours and instead
36 characterizes a material using macro-scale observations. The methodology for applying CDT
37 model consists of three concepts:

- 38 • The elastic-viscoelastic correspondence principle;
- 39 • The definition of a Pseudo-Energy Density Function; and
- 40 • The definition of a Damage Evolution Law.

41 Schapery (1984) suggested that constitutive equations for certain viscoelastic media are
42 identical to those for the elastic cases, but stresses and strains are not necessarily physical
43 quantities in the viscoelastic body [7]. Instead they are pseudo variables in the form of
44 convolution integrals. The pseudo-strain, for instance, is defined as:

$$\varepsilon^R = \frac{1}{E_R} \cdot \int_0^t E(t-\tau) \frac{d\varepsilon}{d\tau} d\tau \quad (1)$$

in which ε^R is the pseudo-strain, E_R is a reference stiffness modulus, ε is the actual strain, $E(t)$ is the relaxation modulus, t is real time and τ is an auxiliary time integration variable.

For the case of bending fatigue tests under harmonic load, the pseudo-strain can be computed as:

$$\varepsilon_0^R = \frac{1}{E_R} [\varepsilon_0 \cdot |S^*| \cdot \sin(\omega t + \theta + \varphi)] \quad (2)$$

in which ε_0 is the imposed strain amplitude, $|S^*|$ is the norm of the complex flexural stiffness, ω is the frequency, θ is a regression constant and φ is the phase angle. Eq. (2) is similar to an expression used by Lee (1996) [9], but here the dynamic flexural stiffness $|S^*|$ is used instead of the uniaxial dynamic modulus $|E^*|$.

The Pseudo-Energy Density Function, may be computed with the following expression for tests subjected to harmonic loads with constant strain amplitude [9]:

$$W^R = \frac{I}{2} \cdot C \cdot (\varepsilon^R)^2 \quad (3)$$

in which W^R is the pseudo-energy density, I is the initial pseudo-stiffness, C is the normalized pseudo-stiffness via dividing the pseudo stiffness by I and ε^R is the pseudo-strain. The pseudo-stiffness is defined as the relationship between the stress (σ) and pseudo-strain (ε^R).

For the general case, W^R is supposed to vary with ε^R and a series of internal damage variables D_s , i.e. these internal damage variables must have their own evolution laws. Park et al. (1996) suggested the following expression for viscoelastic materials [18]:

$$\frac{dD_s}{dt} = \left(-\frac{\partial W^R}{\partial D_s} \right)^{\alpha_s} \quad (4)$$

in which D_s represents a damage variable, the over dot means time rate, D_s is a material constant for each damage variable, and s is the number of damage variables adopted in the model. Summation over repeated indices is not implied in Eq. (4). Here only one internal variable D and one parameter are used to characterize the damage.

Lee (1996) proposed an expression to compute the damage parameter from the results of harmonic tension tests [9]. Daniel (2001) proposed a similar expression, but corrected the time intervals to account only for the time period during which the sample is under tension in haversine loading tests [10]. For bending tests with harmonic loading, as the ones used in this paper, the tension time corresponds to $\frac{1}{2}$ of the total cycle time. Therefore, the expression to obtain the damage parameter from 4PB tests is given by:

$$D = \sum_{i=1}^N \left[\frac{I}{2} \cdot (\varepsilon^R)^2 \cdot (C_{i-1} - C_i) \right]^{\frac{\alpha}{1+\alpha}} \cdot (t_i - t_{i-1})^{\frac{1}{1+\alpha}} \quad (5)$$

1 in which C_i and C_{i-1} are the pseudo-strains for two consecutive data acquisition times t_i and t_{i-1} .
 2 The parameter α is believed to be a viscoelastic property of the material. Some definitions and
 3 correlations can be in related work, but there are still uncertainties determination. The first
 4 definition presented in the literature, relates the value of α to the viscoelastic properties of
 5 material, in particular, the parameter defined in curves of creep or relaxation tests. It was
 6 recommended to correlate α to the slope, m in the central part of the dynamic modulus master
 7 curve [10]. After determination of the internal damage variable D and the parameter α , the
 8 characteristic curve may be modelled. A simple model was proposed by Lee (1996), as follows
 9 [9]:

$$10 \quad C = C_0 - C_1 D^{C_2} \quad (6)$$

11 in which C_i are coefficients determined experimentally. By relating of damage parameter and so-
 12 called pseudo-stiffness parameter of the material during the performance of a fatigue test, one
 13 can observe the evolution of the damage inside it, the C-D curves.

14 3. MATERIALS AND TESTS METHODS

15 Four different types of asphalt-rubber mixtures of gap-graded gradation with different 4th
 16 generation asphalt-rubber compositions added to the CAP 50/70 binder were produced. The
 17 asphalt-rubber blend in which Pellet was used was designated MAB-P, with addition of this
 18 material in 30% of binder mass. The asphalt-rubber blends produced with RAR were named
 19 MAB-R25 (with addition of 25% RAR in binder mass) and MAB-R30-1, MAB-R30-2 and
 20 MAB-R30-3 (with addition of 30% RAR by binder mass). Details of the methodology used
 21 during the production of prismatic samples can be found in Nunes (2017) [19].

22 The 4PB equipment software allows the test configuration according to EN 12967-24 [20],
 23 which allows the execution of a pre-test to obtain the dynamic modulus of the sample. Thus, at
 24 least three prismatic samples for each mixture were tested under the frequency values of 0.1; 0.2;
 25 0.5; 1; 2; 5; 10; 15; 20; 25 Hz and temperatures of 5, 15, 25 and 35 °C for the determination of
 26 the dynamic modulus ($|E^*|$) and phase angle (φ). The imposed deformation was 50 $\mu\epsilon$ in order to
 27 avoid premature damage and represents the external fibers response of the beam to the load. The
 28 sampling rate was determined by the application of sinusoidal pulses, with 200 repetitions per
 29 loading cycle. This step provided information to obtain the master curve of the studied mixtures
 30 that were adjusted according to the sigmoidal model. With the dynamic modulus, the relaxation
 31 and creep modulus can be obtained through some existing forms of conversions. Schapery &
 32 Park (1999) present some approximate formulations that allow the interconversion between the
 33 viscoelastic properties [21].

34 The viscoelastic material properties were estimated through the relaxation modulus
 35 calculation. The relaxation modulus values were calculated for each mixture type using the exact
 36 inter-conversion method. This approach requires the storage modulus E' to allow simple inter-
 37 conversion between the frequency and time domains. Through the Prony series, it is possible to
 38 obtain the relaxation modulus using a Weichert model, with the elements in parallel. The Prony
 39 series function can be expressed using:

$$E(t) = E_\infty + \sum_{i=1}^N E_i \exp^{\frac{-t}{\rho_i}} \quad (7)$$

1 in which E_∞ is the equilibrium relaxation modulus, E_i and ρ_i are the the number of terms of the
2 series of Prony.

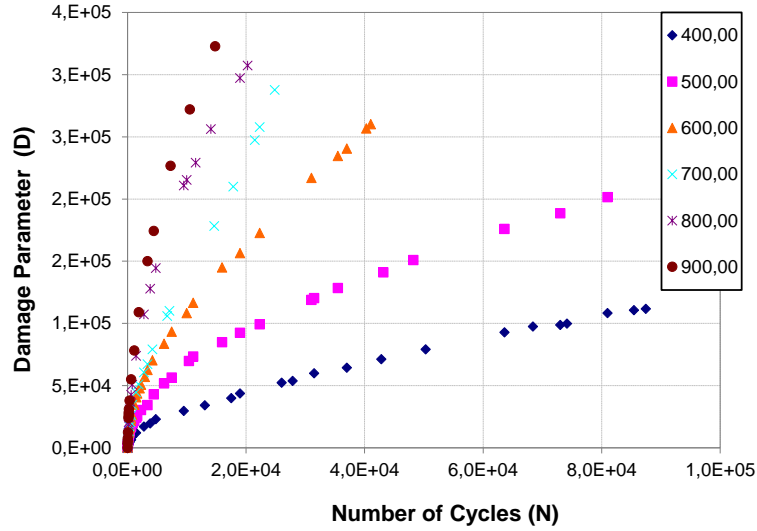
3 The fatigue test was performed according to AASHTO T 321 [17] protocol, using mode of
4 loading set to controlled deformation, and the shape of the sinusoidal pulse varied from 300 to
5 1000 $\mu\epsilon$ at 5 °C; 12.5 °C and 20 °C. In the case of beams the deformation does not appear
6 uniformly along the section and the evolution of the damage progressively occurs from the outer
7 edges toward the center of the sample [9-11]. In this paper, the characteristic curves of the
8 mixtures studied aim to endorse the study of Mello et al. (2010) [16] in the validation for the
9 application of the theory of continuous damage in tests of flexural fatigue.

10 4. RESULTS

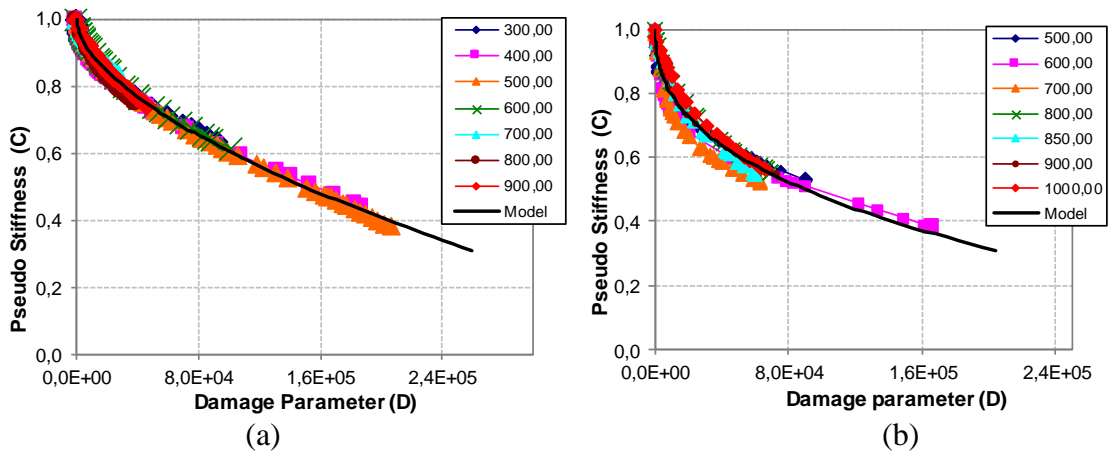
11 Figure 1 shows the evolution of the damage parameter with the number of cycles for
12 different imposed deformation values (in $\mu\epsilon$) for MAB-R30-2 mixture. The curves of cumulative
13 damage versus the number of load cycles show a non-linear damage evolution. The damage rate
14 is higher for the initial cycles of test, then it decreases and becomes almost constant to finally
15 increase again (more obvious in some tests than others), which is indicative of a last stage of
16 failure of the sample. Similar results are obtained for mixtures MAB-P, MAB-R25 and the
17 mixtures MAB-R30-1, MAB-R30-2 and MAB-R30-3, the last three tests in different
18 temperatures. The same tendency is observed, i.e., non-linear damage evolution and faster
19 damage accumulation for higher strain amplitudes. In C-D curves obtained for the mixtures
20 modified with 30% of RAR tested at different temperatures, it was not possible to observe the
21 influence of the test temperature on the evolution of the damage. These results showed similar
22 damage increase rates for the same strain levels.

23 For the calculation of the damage parameter, the previous determination of the value of α is
24 necessary. In this work, the variable α will be determined iteratively, with the aid of the
25 characteristic curve to be determined. With the damage evolution and pseudo-stiffness data
26 throughout all the tests on the beams, an α parameter is determined such that all the curves are
27 superimposed on each other, assuming that such a parameter is a property of the material
28 [9,10,22]. Figure 2 shows the results for the mixtures MAB-P and MAB-R25, for example,
29 subjected to different strain amplitudes, at the temperature of 20°C. A fitting for the
30 Characteristic Curve model, as represented by Eq. (6), is also shown in the same figure. The
31 results were obtained for the seven different strain amplitudes and, nonetheless, all curves of
32 normalized pseudo-stiffness factor versus the damage parameter are practically coincident.
33 Although the results in Figure 2 were obtained from fatigue tests on beam samples under cyclic
34 bending, they agree with the results obtained by other researchers who used uniaxial fatigue tests
35 under tension on cylindrical samples [9-11; 22]. The fitting for Characteristic Curve model
36 obtained in the others mixtures (MAB-R30-1, MAB-R30-2 and MAB-R30-3) also showed good
37 adherence to the model.

38 Table 1 shows the values of parameter α and the parameters of the characteristic curves
39 obtained for all mixtures. The α values obtained for the different mixtures reaffirm the similar
40 behavior of the parameter k_2 , of the traditional model of fatigue, according to the studies carried
41 out in [19].



1
2 **FIGURE 1** Evolution of the damage parameter for different strain amplitudes (MAB-
3 **R30-2, Temp.=20°C; 10 Hz).**



4
5
6 **FIGURE 2** Characteristic curves for different strain amplitudes (a) mix MAB-P,
7 **Temp. = 20°C; 10 Hz (b) mix MAB-R25, Temp. = 20°C; 10 Hz.**

8 **TABLE 1** Coefficients of the characteristic curves for all mixtures.

Mixture	Temperature	C_0	C_1	C_2	α	k_2
MAB-P	20°C	0.99	4.36E-04	0.59	1.91	4.32
MAB-R25	20°C	1.03	7.44E-03	0.37	1.79	5.54
MAB-R30-1	20°C	1.03	3.88E-03	0.42	2.31	5.96
MAB-R30-2	5°C	1.00	1.02E-04	0.67	2.08	3.59
MAB-R30-3	12.5°C	1.01	3.04E-04	0.59	2.30	3.75

9
10 Notice that coefficient C_0 is very close to unity as expected. Coefficient C_1 is closely
11 related to the negative slope of the characteristic curves. Coefficient C_2 is related to the non-
12 linearity of the characteristic curves and it increases for lower temperatures.

13 These behavior of the C-D curves parameters were not similar to the data obtained by
14 Mello et al. (2010) [16]. However, the need for a greater number of flexural fatigue tests for the

1 mixture produced is emphasized, in order to obtain more consistent results, reducing the
2 variability. In addition, it is necessary the numerical simulation stage will allow the analysis of
3 evolution of the damage, according to the characteristic curves, which relate the parameter of
4 damage with the degree of cracking existing inside the material.

5 **5. CONCLUSIONS**

6 The evolution of internal damage in hot mix asphalt (HMA) can be properly evaluated
7 using the framework of the Continuum Damage Theory (CDT) to determine its characteristic
8 curve. The uniqueness of this curve was investigated in this paper by performing fatigue tests in
9 which prismatic samples of HMA were subjected to cyclic bending loads under strain controlled
10 tests. Asphalt mixtures with gap gradation using different 4th asphalt-rubber modified binders
11 were tested and evaluated.

12 The characteristic curves proved to be unique for a wide range of strain amplitudes for all
13 compositions asphalt-rubber mixes investigated. Tests with different temperatures for the same
14 composition (MAB-R30-1, MAB-R30-2 and MAB-R30-3) and the same temperature for
15 different materials (MAB-P, MAB-R25 e MAB-R30-1) showed α parameter with the similar
16 behavior of the parameter k_2 . The C-D curves allows the development of models from numerical
17 tools in the study of the evolution of the damage inside the mixture, which cannot be done when
18 analyzing based on the traditional models of fatigue. Thus, in order to obtain conclusions about
19 the damage evolution in the mixtures produced and on the 4th asphalt-rubber materials,
20 numerical analyzes involving the use of the C-D curves are necessary. In this way, it will
21 facilitate the understanding and definition of solutions capable of assisting in the development of
22 projects closer to reality.
23

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