

# Energy-based thermo-viscoelastic damage model for asphalt concrete mixtures

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## ABSTRACT

In this paper, a three dimensional (3D) analysis framework for temperature-induced damage (thermal cracking) is presented. The analysis framework consists of an energy-based viscoelastic anisotropic damage model for the characterization of damage in asphalt mixtures and a temperature coupling model for the determination of the critical micro-crack initiation threshold (MCIT) and the material damage parameters as a function of temperature. The energy-based analysis framework is implemented in a Finite Element Analysis (FEA) program for the simulation of the Thermal stress restraint specimen test (TSRST). Superpave IDT test is performed to characterize the material viscoelastic and damage properties at -20°C, -10°C and 0°C. The result of the analysis shows the energy-based thermos-viscoelastic damage model is capable of predicting realistic damage behaviour of the asphalt mixture in the TSRST test. The energy-based model exhibits the potential of accounting for the effects of environmental changes (temperature variations) and mechanical loading on the material response in a unified thermodynamic consistent framework.

Keywords: TSRST, thermal cracking, thermo-viscoelastic damage, energy-based viscoelastic damage model, low temperature cracking

## 1. INTRODUCTION

Thermal cracking is one of the primary distress modes that is manifested as transverse cracks on the surface of asphalt pavements. The primary mechanism that drives the development of thermal cracking is the temperature gradient that results due to the severe variations in the climatic condition especially in low temperature climatic regions. The development of thermal cracking is mitigated at high temperature range by the increased capabilities of the mixture to relax stresses. From a micromechanical perspective, the difference in the thermomechanical properties of the asphalt binder and the aggregates together with the imposed boundary condition introduces differential internal constraint that results in tensile stress build-up. Factors that affect the thermal behaviour of asphalt concrete mixtures include binder type, aggregate type and gradation, mixture volumetric properties, thermos-volumetric properties (i.e. coefficient of thermal expansion), the temperature and the cooling rate [1]–[3].

The current specifications for the selection of mixture for thermal cracking resistance asphalt mixtures include a set of experimental tests and limits to check the capabilities of asphalt binder and asphalt mixtures to resist thermal cracking. The experimental setup includes the Bending Beam Rheometer (BBR) and Direct Tension (DT) test for the binder, while the Thermal stress restraint specimen test (TSRST) and the Superpave IDT creep and strength tests are used for asphalt concrete mixture evaluation. The PG grading of asphalt binders is used in characterizing and selecting mixtures for low temperature cracking pavement applications. The national-pooled

fund study identified the fracture energy ( $G_f$ ) from the Semi-Circular Bend (SCB) Test and the binder failure strain as performance indicators for thermal cracking in asphalt concrete mixtures [4], [5].

Different models have been proposed for the characterization of the thermal cracking behaviour of asphalt mixtures under thermal cyclic loading and constant thermal cool rate conditions. Phenomenological models similar to the traditional fatigue analysis approach using the Miner's rule have been used for predicting the number of thermal cycles to failure. The energy release rate  $J_{IC}$  [6] and the  $C^*$  line integral have been used as parameters to evaluate the low temperature cracking potential of asphalt concrete mixtures. Lytton et. al [7] proposed models based on fracture mechanics to predict the number of temperature cycles to thermal fracture. The SHRP research project presented a comprehensive framework (TC-model) that captures the mechanisms governing the thermal cracking in asphalt pavements. The IlliTc-model was developed as part of the National pooled fund study on low temperature cracking [8]. The IlliTc-model relies on the 2D-FEM and a bi-linear cohesive zone modelling technique to account for the initiation and propagation of cracks due to thermal stress build-up. Other models that have been used to characterize the thermal cracking in asphalt mixtures include [9]–[14]. Most of these models are based on 1D and 2D-analysis framework without consideration for the dependency of the material damage parameters on temperature, the anisotropic damage behaviour of Asphalt concrete, the three-dimensional stress distribution, and its effect on the material damage behaviour.

This paper presents and explores the potentials of a unified cracking model that can be used to account for the coupled effects of thermal and mechanical loading conditions on asphalt concrete response in a 3D-analysis framework. The energy-based damage model for viscoelastic asphalt mixtures developed by [15]–[18] has been used to characterize and model the behaviour of AC mixtures due to cracking under mechanical loading conditions. The model is incorporated with a temperature coupling model that accounts for the changes in the material damage parameters due to changes in the temperature. The energy based thermo-viscoelastic damage model is used to predict the material damage behaviour during the TSRST test.

## 2. THERMO-MECHANICAL CONSTITUTIVE RELATIONSHIP

The thermo-elastic boundary value problem considering infinitesimal deformation derived within a thermodynamic framework is presented in [19]. The resulting equations are presented as follows:

- Conservation of linear momentum (neglecting inertia term)

$$\sigma_{ji,j} + \rho f_i = 0 \quad 1.$$

- Strain-displacement relationship

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad 2.$$

- Constitutive equation (neglecting residual stress)

$$\sigma_{ij} = D_{ijkl}(\varepsilon_{kl} - \varepsilon_{kl}^\theta) + \sum_{i=1}^n D_{ijkl}^i(\varepsilon_{kl} - \varepsilon_{kl}^\theta - \varepsilon_{kl}^{v,i}) \quad 3.$$

$$\varepsilon_{kl}^\theta = \beta_{kl}(T - T_o) \quad 4.$$

- Conservation of energy (heat conduction equation)

$$\rho C_v \dot{T} - (k_{ij} T_{,j})_{,i} - \rho \dot{r} = 0 \quad 5.$$

1 where,  $\sigma_{ij}$  is the stress tensor,  $\rho$  is the mass density,  $f_i$  is the body force per unit mass,  $\varepsilon_{ij}$  is the  
2 total strain tensor,  $u_i$  is the deformation field,  $D_{ijkl}$  is the elastic modulus tensor,  $D_{ijkl}^i$  is the elastic  
3 modulus of the  $i^{\text{th}}$  leg of the prony series,  $\varepsilon_{kl}^{\theta}$  is the thermal strain tensor,  $\varepsilon_{kl}^{v,i}$  is the viscous strain  
4 tensor of the  $i^{\text{th}}$  leg of the prony series,  $\beta_{kl}$  is the thermal expansion tensor,  $T$  is the temperature  
5 field,  $T_o$  is the strain reference temperature,  $C_v$  is the specific heat at constant volume,  $k_{ij}$  is the  
6 thermal conductivity tensor, and  $r$  is the heat supply per unit mass.

### 8 3. ENERGY-BASED VISCOELASTIC DAMAGE MODEL

9 Onifade et al. [15]–[17] proposed a viscoelastic anisotropic damage model based on energy  
10 balance with potentials for the identification of the critical threshold for micro-crack initiation and  
11 its consequent evolution based on thermodynamics of irreversible processes and Continuum  
12 Damage Mechanics. A non-associative damage formulation is used to derive different criteria for  
13 damage initiation and evolution. The initiation and evolution of damage is considered only on the  
14 positive part of damage conjugate  $Y_{ij}^+$ . The micro-crack initiation criterion in one principal damage  
15 direction is expressed as follows:

$$f^d = \wp_{1,c}^*(Y^+) - \wp_{1,c}^*(S_o, k_2) - R(r) = 0 \quad 6.$$

16 Where:

- 17  $\wp_{1,c}^*(Y^+)$ : is the micro-crack initiation potential  
18  $\wp_{1,c}^*(S_o, k_2)$ : is the critical micro-crack initiation threshold  
19  $Y^+$ : is a measure of the strain energy density  
20  $S_o$ : is an energy term obtained from a strength test  
21  $R$ : is the damage softening term

22 The micro-crack initiation potential  $\wp_{1,c}^*(Y^+)$  is driven by thermodynamic conjugate of the damage  
23 variable ( $Y^+$ ) and expressed as:

$$\wp_{1,c}^*(Y_{ij}^+) = \frac{S_o}{k_2 + 1} \cdot \left( \frac{\sqrt{Y_{ij}^+ : Y_{ij}^+}}{2 \cdot S_o \cdot (1 - D_{ij})} \right)^{k_2 + 1} \quad 7.$$

24 The critical micro-crack initiation threshold  $\wp_{1,c}^*(S_o, k_2)$  is expressed as:

$$\wp_{1,c}^*(S_o, k_2) = \frac{S_o}{k_2 + 1} \quad 8.$$

25 The micro-crack propagation criterion  $F_D$  in each principal damage direction is used to derive the  
26 evolution of damage and expressed as:

$$F_D = \alpha \cdot \wp_{1,c}^*(Y_{ij}^+) - \alpha \cdot \wp_{1,c}^*(S_o, k_2) - R(r_{ij}) = 0 \quad 9.$$

27 where,  $\alpha$  is  $k_1/k_2$ . The evolution of micro-crack is obtained with respect to the dissipative micro-  
28 crack potential  $F_D$  by taking the derivative of the dissipation potential. The resulting power-law  
29 type damage evolution law is given as:

$$\dot{D}_{ij} = \frac{k_1}{k_2} \cdot \left( \frac{\sqrt{Y_{ij}^+ : Y_{ij}^+} \cdot (I - D_{ij})^{-1}}{2 \cdot S_o} \right)^{k_2} \cdot \frac{Y_{ij}^+}{\sqrt{Y_{ij}^+ : Y_{ij}^+}} : \dot{\epsilon}_{ij} \quad 10.$$

1 where,  $k_1$ ,  $k_2$  and  $S_o$  are material parameters that needs to be determined to together with the linear  
 2 viscoelastic material properties to model the material damage behavior. Details of the damage  
 3 model can be found in [16], [17].

#### 4 Temperature coupling

5 Temperature coupling parameters  $G(T)$ ,  $H(T)$  and  $B(T)$  are introduced to obtain the critical micro-  
 6 cracking damage initiation threshold ( $\phi_{1,c}^*$ ), and the damage parameters  $k_1$  and  $k_2$  at other  
 7 temperatures respectively using the master creep compliance shift factor ( $a_T$ ).

$$\phi_{1,c}^*(T) = \phi_{1,c,ref}^* \cdot G(T), \quad G(T) = \exp(-(\theta_1 \cdot \log(a_T))) \quad 11.$$

$$k_1(T) = k_{1,ref} \cdot H(T), \quad H(T) = \exp(\theta_2 \cdot \log(a_T)) \quad 12.$$

$$k_2(T) = k_{2,ref} \cdot B(T), \quad B(T) = \exp \left[ -\theta_3 \cdot \left( 1 - \frac{T}{T_{ref}} \right) \right] \quad 13.$$

#### 10 Macro-crack formation

11 The Fracture Energy (FE) obtained by means of differential strain gauge observation in the  
 12 Superpave IDT strength test has been used as the energy limit for macro-crack initiation. A power-  
 13 law relationship exist between the damage parameter  $k_1$  and the Fracture Energy (FE):

$$FE = A \cdot k_1^{-n}, \quad 14.$$

14 where, A and n are material constants. The relationship in Eq. (14) presents the possibility of  
 15 predicting the Fracture Energy (FE) over a wide temperature range.

#### 16 4. MATERIAL AND TESTING

17 In this paper, the Superpave IDT test is used to characterize the performance of the asphalt  
 18 concrete mixture. The asphalt mixture was fabricated using unmodified 70/100 penetration grade  
 19 binder and a dense graded crushed granite aggregate with maximum aggregate size of 11mm. The  
 20 Superpave IDT resilient modulus and creep compliance tests are used to obtain the linear  
 21 viscoelastic properties while the Superpave IDT strength test is used to obtain the strength and  
 22 material damage parameters of the asphalt mixture. The procedure used for the interpretation of  
 23 the Superpave IDT strength test for the identification of the material damage parameters is  
 24 presented in [17]. Table 1 shows a summary of the Superpave IDT test results as well as the  
 25 material damage parameters where  $M_r$  is the resilient modulus,  $D_1$  and  $m$  are creep compliance  
 26 parameters (100s creep test),  $\mu$  is the Poisson's ratio, FE is the Fracture Energy,  $k_1$ ,  $k_2$  and  $S_o$  are  
 27 material damage parameters and  $\phi_{1,c}^*$  is the critical micro-crack initiation threshold (MCIT). The  
 28 Poisson's ratio is modelled as a temperature dependent material property using a first order  
 29 polynomial function. Table 2 shows the temperature coupling, WLF shift factor and FE model  
 30 parameters.

31

1 TABLE 1: Summary of Superpave IDT creep and strength test results

Temp	$M_r$ (GPa)	$D_1$ (1/Gpa)	$m$	$\mu$	FE (kJ/m <sup>3</sup> )	$k_1$	$k_2$	$S_0$ (kJ/m <sup>3</sup> )	$\phi_{1,c}^*$ (kJ/m <sup>3</sup> )
-20	20.96	0.0621	0.3951	0.2022	0.52	253.5	1.57	0.177	0.0689
-10	16.7	0.075	0.6156	0.2249	0.97	116.2	0.95	0.169	0.0864
0	10.88	0.8947	0.6254	0.2894	3.47	24.6	0.52	0.264	0.174

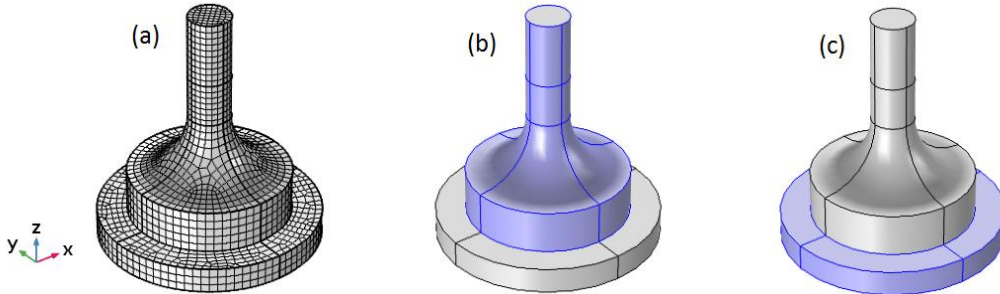
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TABLE 2: Temperature coupling, WLF shift factor and FE model parameters.

Temperature coupling			WLF shift factor			FE model	
$\theta_1$	$\theta_2$	$\theta_3$	C1	C2 (K)	Tref (degC)	A (kJ/m <sup>3</sup> )	n
0.3699	0.7559	1.0621	30.5	250.5	-20	47.66	0.8181

4 **5. TSRST SIMULATION**

5 The energy-based damage model and the temperature coupling model is implemented in  
6 COMSOL Multiphysics ® for the three-dimensional (3D) Finite Element simulation of the  
7 TSRST. The test simulation is carried out on a cylindrical specimen (250mm long) with a thin  
8 central section (50mm diameter) to ensure uniform stress and strain distributions in the central  
9 specimen section and a diameter of 95mm at the base. Figure 1 shows the test geometry and  
10 boundary conditions used in the simulation. The material is cooled from 2°C at a rate of 10°C/hr.  
11 The specific heat capacity, thermal diffusivity and coefficient of thermal expansion are 1000  
12 [J/kgK], 1.86 [W/mK] and 2e-5 [1/K] respectively.

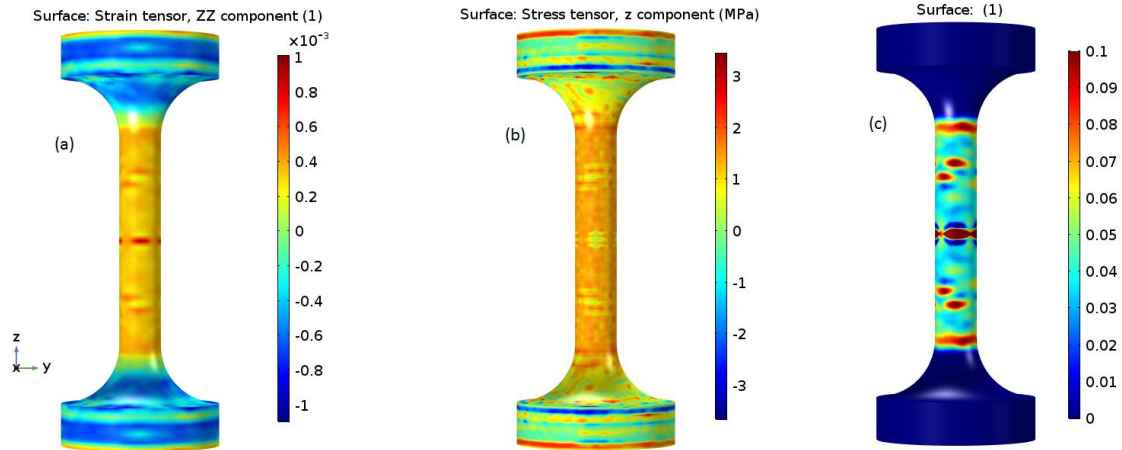


13  
14 FIGURE 1: Finite element mesh and boundary conditions of the simulated TSRST test. (a) FE  
15 mesh, (b) prescribed temperature cooling condition, (c) Fixed domain constraint condition

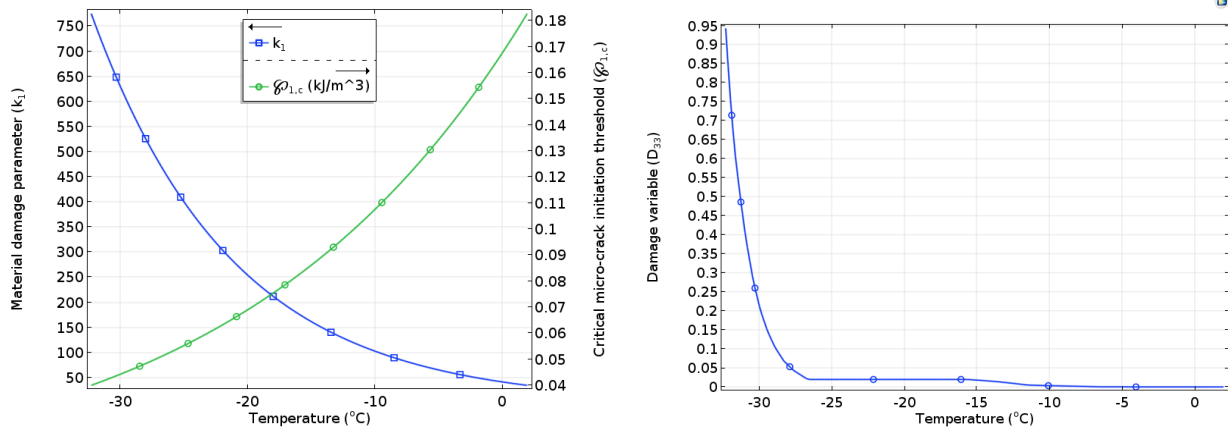
16 **6. RESULTS AND DISCUSSION**

17 The result of the FEM simulation of the TSRST test is shown in Figures 2 and 3. Figure 2  
18 shows the contour plot of the strain, stress and damage density distribution at 32°C. It can be seen  
19 from Figure 2c that the energy-based thermo-viscoelastic damage model is capable of predicting  
20 realistic crack bands and damage patterns of the 3D test specimen. It can be seen from Figure 3a  
21 that the incorporation of the temperature coupling model enables the prediction of the damage  
22 parameter (e.g.  $k_1$ ) and the critical threshold for micro-crack initiation ( $\phi_{1,c}^*$ ) with reasonable  
23 accuracy over a wide range of temperature. Figure 3b shows the evolution of the maximum value  
24 of the damage variable in the 3D specimen. It can be seen from Figure 3b that micro-crack

1 initiation occurs at  $-11^{\circ}\text{C}$ , further cooling didn't result in substantial accumulation of damage due  
 2 to the stress relaxation potentials of the material. It can however be seen that the internal resistance  
 3 to damage accumulation diminishes with further cooling and a rapid accumulation of damage  
 4 occurs as the material reaches  $-26.7^{\circ}\text{C}$ . The experimentally observed fracture temperature in the  
 5 TSRST test is  $-25.7^{\circ}\text{C}$  with a standard deviation of 0.62 [20].  
 6



7  
 8 FIGURE 2: TSRST simulation results at  $32^{\circ}\text{C}$  (a) Strain distribution in z-direction, (b) Stress  
 9 distribution in z-direction, (c) Damage variable in z-direction.



10  
 11 FIGURE 3: (a) Evolution of material damage parameter ( $k_1$ ) and critical micro-crack initiation  
 12 threshold ( $D_c$ ), (b) Evolution of maximum damage density in 3D test geometry

### 13 7. CONCLUSION

14 In this paper, a three-dimensional analysis framework for the evaluation of the thermally  
 15 induced cracking is presented. The framework is developed by incorporating temperature coupling  
 16 model in an energy-based anisotropic damage model to account for the effect of temperature  
 17 variation on the critical threshold for damage initiation, damage parameters and the material  
 18 damage behaviour. The result of the analysis in this paper shows that the energy-based model is  
 19 capable of predicting realistic damage behaviour of the asphalt mixture in the TSRST test. The  
 20 energy-based thermo-viscoelastic damage model presents the potential and the capabilities of  
 21 capturing the coupled effects of thermal and mechanical loading conditions on asphalt concrete  
 22 response in a unified thermodynamic consistent framework.

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