Calibration of a Mechanistic-Empirical Model for the Estimation of the Dynamic Modulus of Asphalt Mixtures

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ABSTRACT

Micromechanical modelling is used to determine the effective properties of heterogeneous materials such as asphalt mixtures, considering the properties of the individual components (asphalt binder, aggregates, voids), their volumetric concentrations and their geometric shapes. In an earlier stage of this research, a mechanistic-empirical model for the estimation of the dynamic modulus $E^*$ of asphalt mixture was developed using micromechanical concepts through a 3D-Spherical model of relative simplicity and based on the valid conceptual frame of the Applied Mechanics. Empirical adjustment factors are then introduced to take into account the complex behaviour of this kind of materials. The inputs of the model are the conventional results obtained during the mix design procedure such as aggregate gradation, volumetric and asphalt binder properties. In order to validate and evaluate the predictive capacity of the developed model, it has been applied to the results compiled in a large database containing the required information of different asphalt mixtures. This work presents a brief description of the developed model and its calibration. The predictive capacity of the model was evaluated by a comparison between measured and estimated dynamic modules values using statistical criteria.

Keywords: Asphalt mixtures, Micromechanics, Dynamic modulus, Predictive model

1. INTRODUCTION

The dynamic modulus $|E^*|$ is the main input material property of asphalt mixtures for the pavement design procedures based on mechanistic principles. It determines the distribution of stress and strains into the pavement structure and also, it can be correlated with the rutting and fatigue cracking behaviour of the bituminous layers [1].

The dynamic modulus $|E^*|$ is determined in laboratory by different procedures but in all cases, it is a time consuming test that require sophisticated equipment and well-trained personnel. When these equipments are not available, the dynamic modulus of the asphalt mixtures could be estimated with different predictive models based on the volumetric properties of the mixture, the aggregate gradations and the binder characteristics using regression analysis from experimental data. Into this kind of predictive models, the Witczak equation in its different versions [2-3] and the Hirsch model [4] are the most used ones.

In a previous paper [5], these estimation models were reviewed considering their advantages and disadvantages and it was concluded that, when testing results are not available, reliable first order dynamic modulus estimates for asphalt mixtures can be obtained. Also, the predictive capabilities of these predictive procedures could be improved using additional information, changing the functional form of the model or calibrating them.

Thus, in order to develop a more robust predictive model of the dynamic modulus of asphalt mixtures, a different point of view was considered developing a mechanistic-empirical model based on micromechanical concepts. Micromechanical modelling is used to determine the effective properties of a heterogeneous material from the inherent properties of the different components, their volumetric concentrations and their geometric shapes. Numerous analytical models have been developed starting from the fundamental work of Eshelby [6] that have been used to investigate a wide range of composite materials.
The Generalized Self-Consistent Scheme [7-9] provides a unique solution for the composite considered as a single sphere embedded in an infinite homogeneous medium with an unknown effective property. Other researchers have developed models built with a combination of multiple phases in series and parallel and introducing empirical factors of adjustment. Finally, several authors [10-11] have developed analytical solutions in 2D applying 3 and 4 layer models and adopting different assumptions of plane state of stresses or strains.

In a previous stage of this research, a 3D-Spherical Model of relative simplicity but based on the valid conceptual frame of the Applied Mechanics, was developed [12]. The inputs of the model are the conventional results obtained during the mix design procedure such as aggregate gradation, volumetric and asphalt binder properties. Subsequently, empirical adjustment factors are added to the model to take into account the complex behaviour of this kind of materials.

After a brief description of the developed model, this paper presents its calibration using the experimental results compiled in a large database containing the required information of different asphalt mixtures. Finally, the predictive capacity of the model is evaluated by a comparison between measured and estimated $|E^*|$ values using statistical criteria.

2. BRIEF DESCRIPTION OF THE 3D-SPHERICAL MODEL

The developed model is a 3D-Spherical Model where each aggregate is considered as a sphere covered by a film of constant thickness of the mastic and embedded in an equivalent homogeneous spherical media with finite size of asphalt mix whose effective properties are unknown as shown in FIGURE 1.

![FIGURE 1 The 3D-Spherical Model](image)

Using the equivalent media concept introduced by Eshelby [6], the outermost layer can be considered as an equivalent media combined with the other two inner layers (aggregate and mastic) and where the properties of this outermost layer are the properties of the asphalt mixture as a micromechanically inhomogeneous material treated as a macromechanically homogeneous composite material. In this model, it was considered that the components are all elastic materials.

In this figure, (a) is the radius of the aggregate, (b-a) is the thickness of the mastic film and (c-b) is the thickness of the equivalent homogeneous media. Also, $E_1$, $E_2$, $E_0(a)$, $\nu_1$, $\nu_2$ and $\nu_0$ are the elastic properties of the aggregate, mastic and homogeneous media respectively. Based on the general concepts of the Applied Mechanics of elastic bodies (isotropic and linear elasticity) for each layer, perfect bonding between neighboring layers, and uniform distribution of the pressure $p$ at the boundary $r=c$, the displacements on each
interface can be obtained by applying the theory of elasticity. Through a laborious mathematical work, the modulus of the homogeneous material can be obtained as:

\[
E_0(a) = \frac{(1-2\nu_0)b}{C \cdot a^2 \cdot (1-J) + D \cdot b^4 - J \cdot a^3 b} \cdot E_1
\]  

(1)

with:

\[
C = \frac{(1+\nu_1)a}{2(b^3 - a^3)} \quad D = \frac{(1-2\nu_1)}{b^3 - a^3} \quad J = \frac{C \cdot b^2 + D \cdot a^3}{F + C \cdot b^2 + D \cdot a^4}
\]  

(2)

Thus, the elastic modulus of the equivalent media \(E_0(a)\) (i.e. the asphalt mixture) can be calculated if the elastic properties of the aggregate and the mastic, the aggregate size and the thickness of the mastic film are known.

3. CONSIDERATIONS FOR A REAL ASPHALT MIXTURE

For a real asphalt mixture, the developed model must be adapted in order to take into account the aggregate gradation, the volumetric content of asphalt binder and the air void distribution assuming the following considerations:

- The mastic is composed with the volume of asphalt binder and aggregates smaller than 0.075 mm
- The aggregates greater than 0.075 mm are spheres covered with a uniform thickness of mastic.
- It was assumed that the asphalt binder behaves as an elastic material for a given condition of temperature and loading frequency. Its elastic modulus (stiffness) was estimated from the A and VTS parameters of the ASTM A-VTS relationship [13].
- The real aggregate gradation was discretized in 17 groups with a single dimension corresponding to their mean diameter. Eq. (1) allows the estimation of the elastic modulus \(E_0(a)\) for a given dimension of the aggregate. Then, the integration of these 17 values according to the volume concentration of each dimensional category results in the final estimated dynamic modulus \(E_0\) of the asphalt mixture.
- The average thickness of the mastic film has been estimated from the mastic content and the percentage and specific surface of each aggregate group assuming that each aggregate particle larger than 0.075 mm is covered by a spherical mastic film of constant thickness.
- The air voids were considered as spherical bubbles embedded in a composite without voids. Then a two-step calculation process has been applied: in the first step, the effective modulus of the equivalent media was estimated by equations (1) and (2); in the second step, this equivalent media wraps the air bubbles to obtain a new elastic modulus of a composite formed by the asphalt mixture and its voids as shown in FIGURE 2.
- According to Castelblanco [14] it was assumed that there is a unique relationship between the air void content in the mixture and the average diameter of the air bubbles as it is shown in FIGURE 3. Finally in the second step, the elastic modulus of the asphalt mixture with air voids can be calculated with the Eqs. (1) and (2) with \(E_1=0\). For the given air void content and diameter of the air bubbles, the thickness of the surrounding shell of asphalt mixture without voids can be calculated as:

\[
t_1 = \frac{a_1}{3 \sqrt[3]{Va \cdot 100}} - a_1
\]  

(3)
where $V_a$ is the air void content; $a_1$ is the radius of the air bubble and $t_1$ is the thickness of the asphalt mixture without voids.

A more detailed description of the model and the consideration for a real asphalt mixture can be found in Ref. [12].

![FIGURE 2 Two-steps Procedure for the Air Voids](image)

The gradation, binder and volumetric properties of each data point was supplied in a sequential and systematic process to the 3D-Spherical Model in order to calculate 2860 estimated $|E^*|$ values to be compared to the experimentally measured ones.

5. OBTAINED RESULTS

![FIGURE 3 Relationship between Air Void Content and Average Diameter](image)

In order to evaluate the predictive capability of the 3D-Spherical Model and the quality of the produced estimations, experimental results reported by Witczak and Mirza [15] at the University of Maryland and by Tashman and Elangovan [16] at the Washington State Transportation Center (TRAC) were considered. This experimental information reported by these researchers are the gradation, volumetric and binder characteristics and measured $|E^*|$ values at different testing frequencies and temperatures for 42 asphalt mixtures made with conventional and modified asphalt binders. All this information was compiled in a large database totaling 2860 sets of data points.

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5. OBTAINED RESULTS

FIGURE 4 shows the comparison between measured ($|E^*|$) and estimated ($E_0$) dynamic modulus values in the log-log space. Also in the same figure, the Line of Equality is shown.
A visual and qualitative evaluation of this figure shows that the points of comparison are tightly located along a straight line indicating that the model is able to produce reasonable estimates for the different asphalt mixtures and testing conditions of frequency and temperature.

However, this distribution of points differs significantly from the Line of Equality, showing that the developed model is well adapted for the higher $|E^*|$ values but it fails for the lower ones. The higher $|E^*|$ values are for the lower temperatures and/or the higher testing frequencies where the mastic is stiffer and it is the main responsible for the mechanical response of the asphalt mixture. On the other hand, the lower $|E^*|$ values are for the higher temperatures and/or the lower testing frequencies where the aggregate interlocking is the main responsible for the mechanical response of the asphalt mixture. Given that the model does not take into account this frictional contribution, this observed behaviour is consequently logical.

Therefore, an empirical calibration factor has been introduced to save the apparent contradiction between the simplicity of the model and the complexity of the mechanical behaviour of the asphalt mixtures.

### 6. MODEL CALIBRATION

An empirical calibration factor $F_c$ has been adjusted as a function of the stiffness of the asphalt binder and the volumetric concentration of aggregates minimizing the differences between measured and estimated $|E^*|$ values using a nonlinear least square regression procedure. The adjusted model results:

$$|E^*| = E_0 \cdot F_c = E_0 \cdot \left(3.838 \cdot \frac{V_g}{V_g + V_b} \cdot G^*^{-0.575} + 0.823\right) \tag{4}$$

where $E_0$ is the estimated dynamic modulus value without adjustment, $V_g$ is the volumetric content of aggregates in $\%, V_b$ is the volumetric content of asphalt binder in $\%$ and $G^*$ is the dynamic shear modulus of the asphalt binder in MPa. According to the functional form of Eq. (4), the magnitude of the calibration factor is more significant when $G^*$ is smaller or the volumetric concentration of aggregates is greater.

FIGURE 5 shows the comparison between measured and estimated $|E^*|$ results for the
To quantitatively evaluate the performance of the estimation model, the quality of the comparisons between measured and estimated results was assessed using goodness-of-fit statistics according to a subjective criteria proposed by Witczak et al. [17] and based on the correlation coefficient \( R^2 \) and the relationship between the Standard error of the estimate values and the Standard deviation of the measured values \( \text{Se/Sy} \).

**FIGURE 5 Measured vs. Estimated |E*| Values for the Calibrated Model**

In the log-log space, the correlation coefficient results \( R^2 = 0.93 \) and the relationship \( \text{Se/Sy} = 0.26 \) showing that the performance of the calibrated model is Excellent in estimating dynamic modulus values for the asphalt mixtures and testing conditions included in the considered database.

**7. CONCLUSIONS**

In this paper, a 3D-Spherical Model for the estimation of the dynamic modulus of asphalt mixtures was calibrated and evaluated. The model is based on the valid conceptual frame of the Applied Mechanics and the inputs are the conventional results obtained during the mix design procedure such as aggregate gradation, volumetric characteristics and asphalt binder properties.

For the application to a real asphalt mixture, several considerations have been introduced regarding the mastic thickness, aggregate gradation and air voids.

An empirical calibration factor \( F_c \) has been adjusted as a function of the stiffness of the asphalt binder and the volumetric concentration of aggregates in order to improve the predicting capability of the model.

The performance of the calibrated model is Excellent in estimating dynamic modulus values for the asphalt mixtures and testing conditions included in the considered database.

It could be concluded that, when testing results are not available, reliable first order dynamic modulus estimates can be obtained using the mechanistic-empirical 3D-model considered in this study.

**8. REFERENCES**


