

1 Calibration of a Mechanistic-Empirical Model for the Estimation of the 2 Dynamic Modulus of Asphalt Mixtures

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5 ABSTRACT

6 Micromechanical modelling is used to determine the effective properties of
7 heterogeneous materials such as asphalt mixtures, considering the properties of the individual
8 components (asphalt binder, aggregates, voids), their volumetric concentrations and their
9 geometric shapes. In an earlier stage of this research, a mechanistic-empirical model for the
10 estimation of the dynamic modulus E^* of asphalt mixture was developed using
11 micromechanical concepts through a 3D-Spherical model of relative simplicity and based on
12 the valid conceptual frame of the Applied Mechanics. Empirical adjustment factors are then
13 introduced to take into account the complex behaviour of this kind of materials. The inputs of
14 the model are the conventional results obtained during the mix design procedure such as
15 aggregate gradation, volumetric and asphalt binder properties. In order to validate and
16 evaluate the predictive capacity of the developed model, it has been applied to the results
17 compiled in a large database containing the required information of different asphalt
18 mixtures. This work presents a brief description of the developed model and its calibration.
19 The predictive capacity of the model was evaluated by a comparison between measured and
20 estimated dynamic modulus values using statistical criteria.

21 **Keywords:** Asphalt mixtures, Micromechanics, Dynamic modulus, Predictive model

22 1. INTRODUCTION

23 The dynamic modulus $|E^*|$ is the main input material property of asphalt mixtures for
24 the pavement design procedures based on mechanistic principles. It determines the
25 distribution of stress and strains into the pavement structure and also, it can be correlated with
26 the rutting and fatigue cracking behaviour of the bituminous layers [1].

27 The dynamic modulus $|E^*|$ is determined in laboratory by different procedures but in
28 all cases, it is a time consuming test that require sophisticated equipment and well-trained
29 personnel. When these equipments are not available, the dynamic modulus of the asphalt
30 mixtures could be estimated with different predictive models based on the volumetric
31 properties of the mixture, the aggregate gradations and the binder characteristics using
32 regression analysis from experimental data. Into this kind of predictive models, the Witczak
33 equation in its different versions [2-3] and the Hirsch model [4] are the most used ones.

34 In a previous paper [5], these estimation models were reviewed considering their
35 advantages and disadvantages and it was concluded that, when testing results are not
36 available, reliable first order dynamic modulus estimates for asphalt mixtures can be
37 obtained. Also, the predictive capabilities of these predictive procedures could be improved
38 using additional information, changing the functional form of the model or calibrating them.

39 Thus, in order to develop a more robust predictive model of the dynamic modulus of
40 asphalt mixtures, a different point of view was considered developing a mechanistic-
41 empirical model based on micromechanical concepts. Micromechanical modelling is used to
42 determine the effective properties of a heterogeneous material from the inherent properties of
43 the different components, their volumetric concentrations and their geometric shapes.
44 Numerous analytical models have been developed starting from the fundamental work of
45 Eshelby [6] that have been used to investigate a wide range of composite materials.

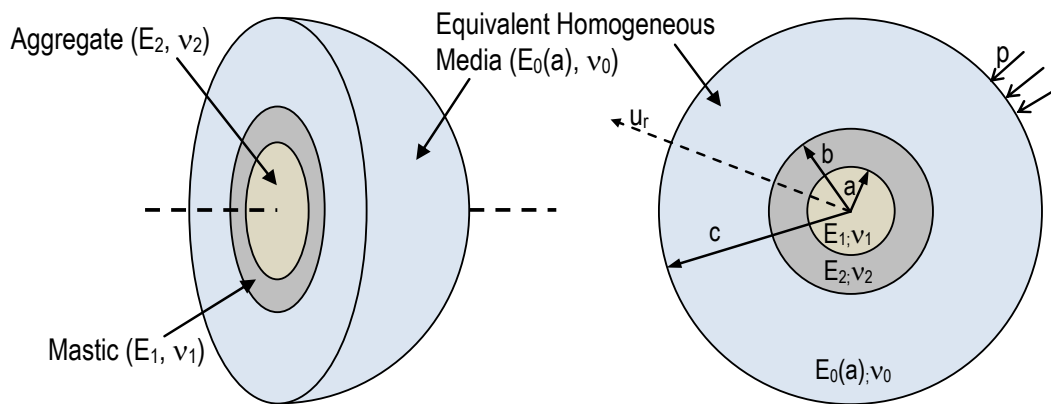
1 The Generalized Self-Consistent Scheme [7-9] provides a unique solution for the
 2 composite considered as a single sphere embedded in an infinite homogeneous medium with
 3 an unknown effective property. Other researchers have developed models built with a
 4 combination of multiple phases in series and parallel and introducing empirical factors of
 5 adjustment. Finally, several authors [10-11] have developed analytical solutions in 2D
 6 applying 3 and 4 layer models and adopting different assumptions of plane state of stresses or
 7 strains.

8 In a previous stage of this research, a 3D-Spherical Model of relative simplicity but
 9 based on the valid conceptual frame of the Applied Mechanics, was developed [12]. The
 10 inputs of the model are the conventional results obtained during the mix design procedure
 11 such as aggregate gradation, volumetric and asphalt binder properties. Subsequently,
 12 empirical adjustment factors are added to the model to take into account the complex
 13 behaviour of this kind of materials.

14 After a brief description of the developed model, this paper presents its calibration
 15 using the experimental results compiled in a large database containing the required
 16 information of different asphalt mixtures. Finally, the predictive capacity of the model is
 17 evaluated by a comparison between measured and estimated $|E^*|$ values using statistical
 18 criteria.

19 2. BRIEF DESCRIPTION OF THE 3D-SPHERICAL MODEL

20 The developed model is a 3D-Spherical Model where each aggregate is considered as
 21 a sphere covered by a film of constant thickness of the mastic and embedded in an equivalent
 22 homogeneous spherical media with finite size of asphalt mix whose effective properties are
 23 unknown as shown in FIGURE 1.



24
 25 **FIGURE 1 The 3D-Spherical Model**

26 Using the equivalent media concept introduced by Eshelby [6], the outermost layer
 27 can be considered as an equivalent media combined with the other two inner layers
 28 (aggregate and mastic) and where the properties of this outermost layer are the properties of
 29 the asphalt mixture as a micromechanically inhomogeneous material treated as a
 30 macromechanically homogeneous composite material. In this model, it was considered that
 31 the components are all elastic materials.

32 In this figure, (a) is the radius of the aggregate, (b-a) is the thickness of the mastic
 33 film and (c-b) is the thickness of the equivalent homogeneous media. Also, E_1 , E_2 , $E_0(a)$, v_1 ,
 34 v_2 and v_0 are the elastic properties of the aggregate, mastic and homogeneous media
 35 respectively. Based on the general concepts of the Applied Mechanics of elastic bodies
 36 (isotropic and linear elasticity) for each layer, perfect bonding between neighboring layers,
 37 and uniform distribution of the pressure p at the boundary $r=c$, the displacements on each

1 interface can be obtained by applying the theory of elasticity. Through a laborious
2 mathematical work, the modulus of the homogeneous material can be obtained as:

$$E_0(a) = \left\{ \frac{(1-2\nu_0)b}{C \cdot a^2 \cdot (1-J) + D(b^4 - J \cdot a^3 b)} \right\} \cdot E_1 \quad (1)$$

3 with:

$$C = \frac{(1+\nu_1)ab}{2(b^3 - a^3)} \quad D = \frac{(1-2\nu_1)}{(b^3 - a^3)} \quad (2)$$

$$J = \frac{Cb^2 + Db^3a}{E_1/E_2 \cdot F + Cb^2 + Da^4} \quad F = (1-2\nu_2)a$$

4 Thus, the elastic modulus of the equivalent media $E_0(a)$ (i.e. the asphalt mixture) can
5 be calculated if the elastic properties of the aggregate and the mastic, the aggregate size and
6 the thickness of the mastic film are known.

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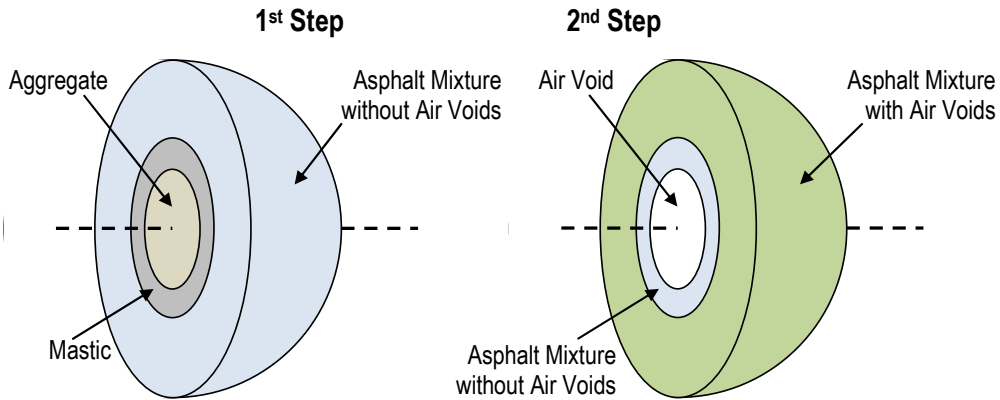
3. CONSIDERATIONS FOR A REAL ASPHALT MIXTURE

10 For a real asphalt mixture, the developed model must be adapted in order to take into
11 account the aggregate gradation, the volumetric content of asphalt binder and the air void
12 distribution assuming the following considerations:

- 13 - The mastic is composed with the volume of asphalt binder and aggregates smaller than
14 0.075 mm
- 15 - The aggregates greater than 0,075 mm are spheres covered with a uniform thickness of
16 mastic.
- 17 - It was assumed that the asphalt binder behaves as an elastic material for a given condition of
18 temperature and loading frequency. Its elastic modulus (stiffness) was estimated from the A
19 and VTS parameters of the ASTM A-VTS relationship [13].
- 20 - The real aggregate gradation was discretized in 17 groups with a single dimension
21 corresponding to their mean diameter. Eq. (1) allows the estimation of the elastic modulus
22 $E_0(a)$ for a given dimension of the aggregate. Then, the integration of these 17 values
23 according to the volume concentration of each dimensional category results in the final
24 estimated dynamic modulus E_0 of the asphalt mixture.
- 25 - The average thickness of the mastic film has been estimated from the mastic content and the
26 percentage and specific surface of each aggregate group assuming that each aggregate
27 particle larger than 0.075 mm is covered by a spherical mastic film of constant thickness.
- 28 - The air voids were considered as spherical bubbles embedded in a composite without voids.
29 Then a two-step calculation process has been applied: in the first step, the effective modulus
30 of the equivalent media was estimated by equations (1) and (2); in the second step, this
31 equivalent media wraps the air bubbles to obtain a new elastic modulus of a composite
32 formed by the asphalt mixture and its voids as shown in FIGURE 2.
- 33 - According to Castelblanco [14] it was assumed that there is a unique relationship between
34 the air void content in the mixture and the average diameter of the air bubbles as it is shown
35 in FIGURE 3. Finally in the second step, the elastic modulus of the asphalt mixture with air
36 voids can be calculated with the Eqs. (1) and (2) with $E_1=0$. For the given air void content
37 and diameter of the air bubbles, the thickness of the surrounding shell of asphalt mixture
38 without voids can be calculated as:

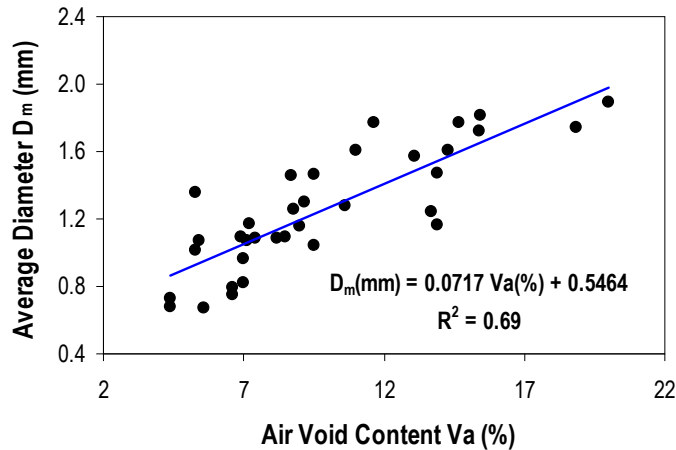
$$t_1 = \frac{a_1}{\sqrt[3]{(V_a/100)}} - a_1 \quad (3)$$

1 where V_a is the air void content; a_1 is the radius of the air bubble and t_1 is the thickness of the
 2 asphalt mixture without voids.



3
 4 **FIGURE 2 Two-steps Procedure for the Air Voids**

5 A more detailed description of the model and the consideration for a real asphalt mixture
 6 can be found in Ref. [12].



7
 8 **FIGURE 3 Relationship between Air Void Content and Average Diameter**

9
 10 **4. MATERIALS AND PROCEDURES**

11
 12 In order to evaluate the predictive capability of the 3D-Spherical Model and the
 13 quality of the produced estimations, experimental results reported by Witczak and Mirza [15]
 14 at the University of Maryland and by Tashman and Elangovan [16] at the Washington State
 15 Transportation Center (TRAC) were considered. This experimental information reported by
 16 these researchers are the gradation, volumetric and binder characteristics and measured $|E^*|$
 17 values at different testing frequencies and temperatures for 42 asphalt mixtures made with
 18 conventional and modified asphalt binders. All this information was compiled in a large
 19 database totalizing 2860 sets of data points.

20 The gradation, binder and volumetric properties of each data point was supplied in a
 21 sequential and systematic process to the 3D-Spherical Model in order to calculate 2860
 22 estimated $|E^*|$ values to be compared to the experimentally measured ones.

23
 24 **5. OBTAINED RESULTS**

25
 26 FIGURE 4 shows the comparison between measured ($|E^*|$) and estimated (E_0) dynamic
 27 modulus values in the log-log space. Also in the same figure, the Line of Equality is shown.

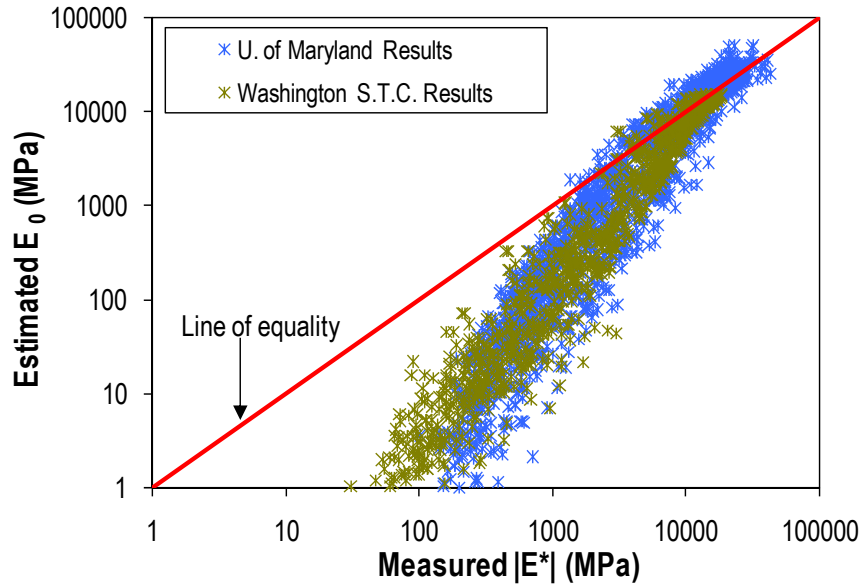


FIGURE 4 Comparison of Measured and Estimated $|E^*|$ Values

A visual and qualitative evaluation of this figure shows that the points of comparison are tightly located along a straight line indicating that the model is able to produce reasonable estimates for the different asphalt mixtures and testing conditions of frequency and temperature.

However, this distribution of points differs significantly from the Line of Equality, showing that the developed model is well adapted for the higher $|E^*|$ values but it fails for the lower ones. The higher $|E^*|$ values are for the lower temperatures and/or the higher testing frequencies where the mastic is stiffer and it is the main responsible for the mechanical response of the asphalt mixture. On the other hand, the lower $|E^*|$ values are for the higher temperatures and/or the lower testing frequencies where the aggregate interlocking is the main responsible for the mechanical response of the asphalt mixture. Given that the model does not take into account this frictional contribution, this observed behaviour is consequently logical.

Therefore, an empirical calibration factor has been introduced to save the apparent contradiction between the simplicity of the model and the complexity of the mechanical behaviour of the asphalt mixtures.

6. MODEL CALIBRATION

An empirical calibration factor F_c has been adjusted as a function of the stiffness of the asphalt binder and the volumetric concentration of aggregates minimizing the differences between measured and estimated $|E^*|$ values using a nonlinear least square regression procedure. The adjusted model results:

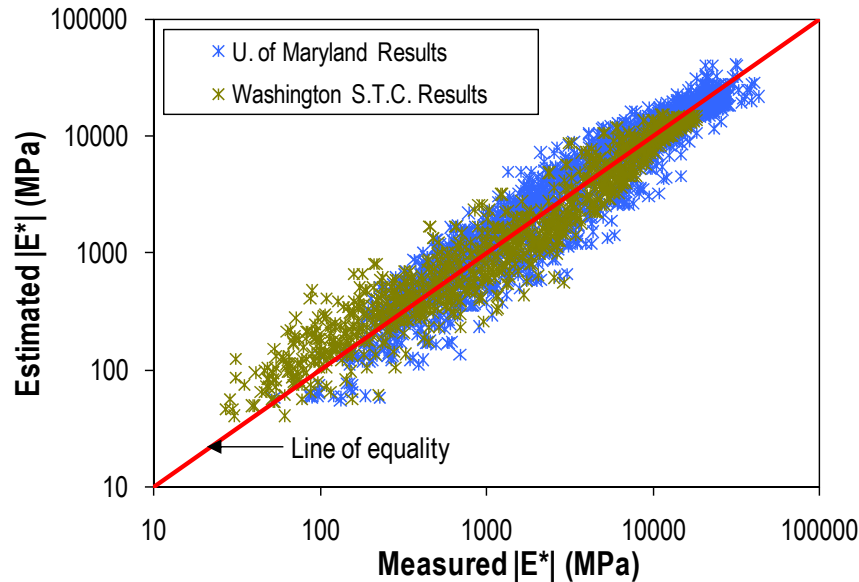
$$|E^*| = E_0 \cdot F_c = E_0 \cdot \left(3.838 \cdot \frac{V_g}{V_g + V_b} \cdot G^{*-0.575} + 0.823 \right) \quad (4)$$

where E_0 is the estimated dynamic modulus value without adjustment, V_g is the volumetric content of aggregates in %, V_b is the volumetric content of asphalt binder in % and G^* is the dynamic shear modulus of the asphalt binder in MPa. According to the functional form of Eq. (4), the magnitude of the calibration factor is more significant when G^* is smaller or the volumetric concentration of aggregates is greater.

FIGURE 5 shows the comparison between measured and estimated $|E^*|$ results for the

1 calibrated model.

2 To quantitatively evaluate the performance of the estimation model, the quality of the
3 comparisons between measured and estimated results was assessed using goodness-of-fit
4 statistics according to a subjective criteria proposed by Witczak et al. [17] and based on the
5 correlation coefficient R^2 and the relationship between the Standard error of the estimate
6 values and the Standard deviation of the measured values Se/Sy .



7
8 **FIGURE 5 Measured vs. Estimated $|E^*|$ Values for the Calibrated Model**

9 In the log-log space, the correlation coefficient results $R^2 = 0.93$ and the relationship
10 $Se/Sy = 0.26$ showing that the performance of the calibrated model is Excellent in estimating
11 dynamic modulus values for the asphalt mixtures and testing conditions included in the
12 considered database.

13 14 7. CONCLUSIONS

15
16 In this paper, a 3D-Spherical Model for the estimation of the dynamic modulus of
17 asphalt mixtures was calibrated and evaluated. The model is based on the valid conceptual
18 frame of the Applied Mechanics and the inputs are the conventional results obtained during
19 the mix design procedure such as aggregate gradation, volumetric characteristics and asphalt
20 binder properties.

21 For the application to a real asphalt mixture, several considerations have been
22 introduced regarding the mastic thickness, aggregate gradation and air voids.

23 An empirical calibration factor F_c has been adjusted as a function of the stiffness of
24 the asphalt binder and the volumetric concentration of aggregates in order to improve the
25 predicting capability of the model.

26 The performance of the calibrated model is Excellent in estimating dynamic modulus
27 values for the asphalt mixtures and testing conditions included in the considered database.

28 It could be concluded that, when testing results are not available, reliable first order
29 dynamic modulus estimates can be obtained using the mechanistic-empirical 3D-model
30 considered in this study.

31 8. REFERENCES

32 [1] NCHRP 1-37a. Mechanistic-Empirical design of new and rehabilitated pavement

1 structures. Draft Report. National Research Council, Washington DC. 2004.

2 [2] Andrei D., Witczak M. and Mirza W. Development of a revised predictive model
3 for the dynamic (complex) modulus of asphalt mixtures. Design Guide for New &
4 Rehabilitated Pavements. Appendix CC-4. NCHRP Project 1-37A National Research
5 Council, 1999.

6 [3] Bari, J. and Witczak M. Development of a new revised version of the Witczak E*
7 predictive model for hot mix asphalt mixtures. Journal of the Association of Asphalt Paving
8 Technologists Vol. 75: 381-423. 2006.

9 [4] Christensen, D.W., Pellinen, T.K. and Bonaquist, R.F. Hirsch model for
10 estimating the modulus of asphalt concrete. Journal of the Association of Asphalt Paving
11 Technologists Volume 72. 2003.

12 [5] Martinez, F. and Angelone, S. Evaluation of different predictive dynamic
13 modulus models of asphalt mixtures used in Argentina. 8th International Conference on the
14 Bearing Capacity of Roads, Railways, and Airfields, BCR2A'09, Urbana – Champaign. 2009.

15 [6] Eshelby, J. D. The determination of the elastic field of an ellipsoidal inclusion
16 and related problems. Proceedings Royal Society, Serie A, No. 241. pp.376-396. 1957.

17 [7] Buttlar, W. G. and Roque, R. Evaluation of empirical and theoretical models to
18 determine asphalt mixture stiffnesses at low temperatures". Journal of the Association of
19 Asphalt Paving Technologists, Vol. 65. 1996.

20 [8] Christensen, R.M. and Lo, K. H. Solutions for effective shear properties in three
21 phase sphere and cylinder models. Journal of the Mechanics and Physics of Solids, Volume
22 27, Issue 4, pp. 315–330. 1999.

23 [9] Shashidhar, N. and Shenoy, A. On Using Micromechanical Models to Describe
24 the Dynamic Mechanical Behavior of Asphalt Mastics. Proceedings of the 79th Annual
25 Meeting of the Transportation Research Board, Washington D. C., 2000.

26 [10] Buttlar, W. G., Bozkurt, D., Al-Khateeb, G. G. and Waldhoff, A. S.
27 Understanding asphalt mastic behavior through micromechanics. Proceedings of the 78th
28 Annual Meeting of the Transportation Research Board, Washington D. C., 1999.

29 [11] Li, G., Li, Y., Metcalf, J. B. and Pang, S. Elastic modulus prediction of asphalt
30 concrete. Journal of Materials in Civil Engineering, Vol. 11, No. 3, August, 1999.

31 [12] Angelone, S., Cauhape Casaux, M. and Martinez, F. Validacion de un modelo
32 micromecanico de estimacion del modulo dinamico de mezclas asfalticas. Proceedings of the
33 XVII Congreso Argentino de Vialidad y Transito, Rosario, Argentina. 2016. (In Spanish).

34 [13] American Society of Testing and Materials. D2493-01 Standard Viscosity-
35 Temperature chart for asphalts". Volume 04.03. 2009.

36 [14] Castelblanco, A. Probabilistic analysis of air void structure and its relationship
37 to permeability and moisture damage of HMA. MSc. Thesis, Texas A&M University, 2004.

38 [15] Witczak, M. and Mirza, W. Dynamic Modulus Database, University of
39 Maryland, 1999.

40 [16] Tashman, L. and Elangovan, M. A. Dynamic Modulus Test – Laboratory
41 investigation and future implementation in the State of Washington. Report No. WARD
42 704.1, Washington State Transportation Center (TRAC), 2007.

43 [17] Witczak, M., Pellinen T. and El-Basyouny M. Pursuit of the simple performance
44 test for asphalt concrete fracture/cracking. Proceedings of the Association of Asphalt Paving
45 Technologists. Vol. 71: pp. 767-778. 2002.