# Algorithm for Determining the Damage Characteristic Curve of Asphalt Mixture

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## ABSTRACT

Fatigue damage in asphalt pavements results mostly from repeated traffic loads. The viscoelastic continuum damage (VECD) model can describe fatigue behavior of asphalt concrete mixtures from fundamental mechanistic principles. This paper presents the algorithm for calculating the damage characteristic curve obtained in direct tension tests taking into account three different types of solicitation. The VECD formulation is presented in a summarized form for the algorithm, in which the pseudo strain, at the instants associated to the observed stress, is calculated using the expression of the linear viscoelasticity stress under controlled strain testing.

**Keywords**: Fatigue, Computational Simulation, Damage curve.

## 1. INTRODUCTION

Fatigue cracking is one of the major distresses in asphalt concrete pavements. The cause of these cracks, which are influenced by repeated (i.e., cyclic) loading over time can be tied to weak pavement sublayers, insufficiently designed asphalt materials, or changes in strain tolerance of the mixture due to field aging.

Continuum Damage Mechanics ignores specific microscale details and attempts to characterize the material by considering the effect of microstructural changes on observable macro properties [3]. It considers the effect of damage in the constitutive modelling of viscoelastic materials by quantifying two variables: (i) damage parameter, S; and (ii) pseudo stiffness, C. The damage parameter, in general, quantifies any change in the microstructure that results in a reduction of stiffness [2].

Kim (2009) [2] developed an algorithm for interpolating the damage curve of asphalt mixtures from the stresses observed in controlled strain tests carried out on laboratory specimens. The algorithm presented herein is an adaptation of the referred algorithm, as follows:

- The original algorithm proposes to arbitrate the increment of damage, while the present algorithm proposes to arbitrate the initial damage, $S_0$;
- The original algorithm calculates $\frac{dC}{dS} = \frac{\Delta C}{\Delta S}$, incrementally, whereas the proposed algorithm calculates $\frac{dC}{dS}$ by the derivative of the Equation 2;
- The proposed algorithm calculates the pseudo strains from the theoretical responses in stresses due to applied controlled strain. This avoids the need to measure the strains in the specimen during the test, facilitating the computation of the pseudo strains.

## 2. LITERATURE REVIEW

### 2.1. Viscoelastic Continuum Damage Theory

The damage theory used in this research, originally developed for elastic materials, is generalized for viscoelastic materials using the elastic-viscoelastic correspondence principle [2]. The work potential theory [4] specifies an internal state variable, S, to quantify damage. This internal state variable quantifies any microstructural changes that result in the observed stiffness reduction. For asphalt concrete under tension stress, this variable is related primarily to the microcracking phenomenon.

The relationship between damage, S, and the normalized pseudo secant modulus, C, is known as the damage characteristic relationship and it is a material function independent of loading conditions [2]. With these considerations, the nonlinear constitutive relationships are given by Equation 1 for stresses.

$$\sigma = C(S)\,\varepsilon^R \qquad (1)$$

According to Kim (2009) [2], the expression for $C = C(S)$ can be represented by a power law as in Equation 2:

$$C(S) = 1 - a\,S^b \qquad (2)$$

Where $a$ and $b$ are regression constants that interpolate the curves obtained in the fatigue tests of the material. These constants characterize the material with respect to its susceptibility to damage.

Substituting $\frac{dC}{dS}$ into the Work Potential Equation by the derivative of Equation 2, and $\frac{dS}{dt}$ by its incremental form, $\frac{\Delta S}{\Delta t}$, one obtains:

$$S_{n+1} = S_n + \left[\tfrac{1}{2}\,a\,b\,S_n^{(b-1)}\left(\varepsilon_n^R\right)^2\right]^{\alpha} \Delta t \qquad (3)$$

Where, $n$ and $n+1$ are the steps at times $t_n$ and $t_{n+1}$, respectively; $S_n$ and $S_{n+1}$ are parameters that quantify the damage at $n$ and $n+1$, respectively; $\Delta t$ is the time interval between $t_n$ and $t_{n+1}$.

Equation 3 can be used to determine the C *vs* S curve, from data obtained in laboratory tests, where $C_{n+1}$ is the integrity of the material at the instant $t_{n+1}$, given by Equation 4.

$$C_{n+1} = \frac{\sigma_{n+1}}{\varepsilon_{n+1}^R} \qquad (4)$$

Where, $\sigma_{n+1}$ is the stress observed at time $t_{n+1}$, and $\varepsilon_{n+1}^R$ is the pseudo strain at time $t_{n+1}$.

## 3. METHODOLOGY

Viscoelastic damage characterization refers to the development of the characteristic damage relationship (i.e., the C *vs* S curve). Although such characterization can be performed under any loading condition, the simplest method is the constant crosshead rate test. This paper presents the algorithm for calculating the C *vs* S curve obtained in direct tension tests considering three different types of solicitation, as indicated ahead.

### 3.1. Constant Crosshead Monotonic Rate Test – CCMRT

Figure 1 shows the CCMRT given by Equations 5 and 6.

$$\varepsilon\ (t) =\ K_0\ t \tag{5}$$

$$\sigma\ (t) =\ K_0 \left[E_\infty\ t \sum_{j=1}^{n} E_j\ \rho_j \left(1 - e^{\frac{-t}{\rho_j}}\right)\right] \tag{6}$$

Where, $K_0 = \frac{dS}{dt}$ = constant is the strain rate of change with time, given by the slope of the line.

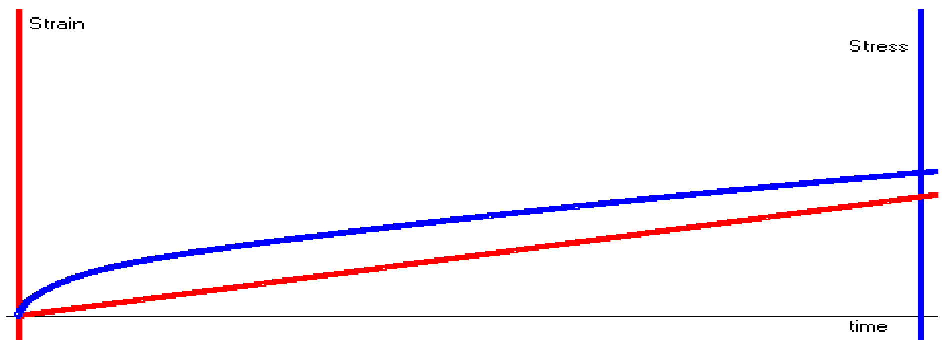


Figure 1: Constant Crosshead Monotonic Rate Test – Source: Authors.

### 3.2. Sinusoidal Controlled Strain Rate Test

For a sinusoidal controlled strain solicitation (Equation 7), the stress response is given by Equation 8. The test is shown schematically in Figure 2.

$$\varepsilon\ (t) =\ \varepsilon_0\ sen\ (wt) \tag{7}$$

$$\sigma\ (t) =\ \varepsilon_0 \left[E_\infty\ sen\ (wt) + w^2 \sum_{j=1}^{n} E_j\ \rho_j^2 \left(sen\ (wt) + \frac{\cos\ (wt)}{w\rho_j} - \frac{e^{\frac{-t}{\rho_j}}}{w\rho_j}\right)\right] \tag{8}$$

Where, $w$ is the angular frequency of the solicitation, and $\varepsilon_0$ is the corresponding strain amplitude.

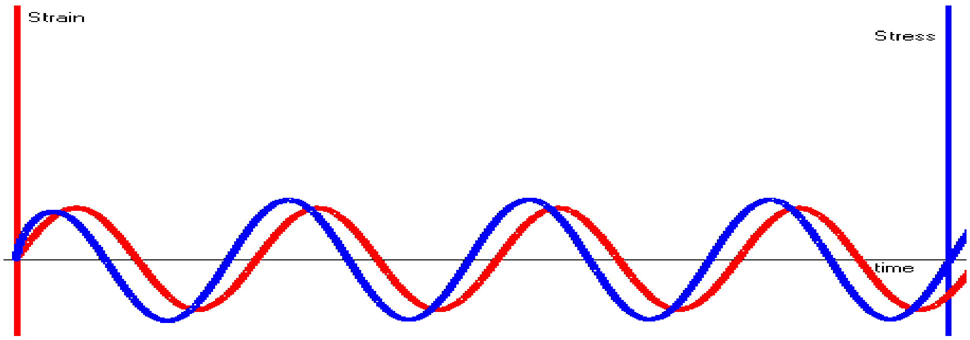


Figure 2: Sinusoidal Controlled Strain Rate Test – Source: Authors.

### 3.3. Controlled Strain Rate Test

For a controlled strain solicitation (Equation 9), the stress response is given by Equation 10. The test is shown schematically in Figure 3.

$$\varepsilon\,(t) = \frac{\varepsilon_0}{2}\left[1 + sen\,(wt - \frac{\pi}{2})\right] \tag{9}$$

$$\sigma\,(t) = \frac{\varepsilon_0}{2}\left\{E_\infty\left[1 + sen\left(wt - \frac{\pi}{2}\right)\right] + w^2\sum_{j=1}^{n}\frac{E_j\,\rho_j^2}{(1 + w^2\rho_j^2)}\left(sen\left(wt - \frac{\pi}{2}\right) + \frac{\cos\,(wt - \frac{\pi}{2})}{w\rho_j} - \frac{e^{\frac{-t}{\rho_j}}}{w\rho_j}\right)\right\} \tag{10}$$

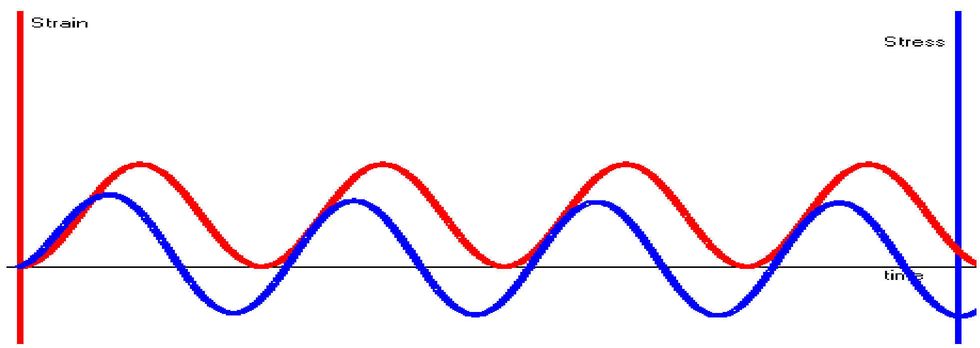


Figure 3: Controlled Strain Rate Test – Source: Authors.

### 3.4. Algorithm to Obtain Parameters $a$ and $b$ of the C *vs* S curve

The proposed algorithm is based on Equations 2, 3, and 4. The algorithm consists of 7 major steps:

Step 1: Values for parameters $a$ and $b$ of the C *vs* S curve are arbitrated $(0 < a < 1)$ and $(0 < b < 1)$.

Step 2: Parameters $C$ and $S$ are taken at $t = 0$. Where $S_0$ = $10^{-6}$ represents the damage inherent in the process of producing the specimen.

Step 3: $S_{n+1}$ is calculated with the value of $Sn$ from the previous time. The $\varepsilon_n^R$ is obtained, depending on the type of test, with Equation 6, 8 or 10.

Step 4: $C_{n+1}$ is calculated in accordance with the stress values obtained from the test. The $\varepsilon_{n+1}^{R}$ is obtained, depending on type of test, with the Equations 6, 8 or 10.

Step 5: New values for $a$ and $b$ are interpolated using Equation 2.

Step 6: If $(\frac{a^{(k+1)} - a^{(k)}}{a^{(k)}} > error)$ and $(\frac{b^{(k+1)} - b^{(k)}}{b^{(k)}} > error)$, then return to Step 2, and the procedure is repeated with the new values of $a$ and $b$.

Step 7: Otherwise, the procedure ends with the values of $a$ and $b$ of the last iteration.

After completing Step 4, a set of ordered pairs $(S^k, C^k)$ should be interpolated using Equation 2, obtaining new values for the following iteration: $a^{(k+1)}$ and $b^{(k+1)}$. Then, the algorithm returns to Step 2 with the new values of $a$ and $b$, repeating the procedure. The value of $\alpha$ is related to the viscoelastic characteristics of the material established by its creep flow or its relaxation modulus. Given the value of $\alpha$, the parameters of the material damage curve can be obtained from the stresses observed in the fatigue test. The stopping criterion is when the relative error between the values of $a$ and $b$ between two consecutive iterations is less than or equal to an acceptable arbitrary error. For the purpose of this paper, the maximum relative error is assumed $10^{-8}$. In the proposed algorithm, the solicitation history is represented by the pseudo strain, $\varepsilon_n^R = \frac{\sigma(t_n)}{E_R}$.

## 4. RESULTS

The algorithm presented in this work has been verified based on experimental data from the literature [1]. The damage was simulated in a material whose Prony series and C *vs* S parameters are known, and then the parameter values were obtained following the procedure described in Section 3.4, considering the three types of solicitation described in Sections 3.1, 3.2, and 3.3. The constants of the Prony series of the material are shown in Table 1, the parameters of the C *vs* S are $a = 5.41 \times 10^{-4}, b = 6.33 \times 10^{-4}$, and $\alpha = 2.993$. The loading data are: strain amplitude $\varepsilon_0 = 0.1$; running time t = 200s; and frequencies of 0.1, 0.5, 1.0, and 2.0 Hz.

The results obtained for the values of $a$ and $b$ coincided with the original values as shown in Table 2, which also presents the number of iterations necessary for the response to be obtained. The results are the same, regardless of the type of solicitation and the frequency considered, which validates the use of Equations 6, 8, and 10. High values of strains were chosen so that the damage manifested itself in a small number of solicitation pulses.

Table 1: Relaxation modulus (kPa) – Source: Babadopulos *et al* (2016).

| $E\infty = 6$ | | |
|---|---|---|
| i | $\rho_i$ | $E_i$ |
| 1 | $1.0 \times 10^{-7}$ | $1.62 \times 10^{2}$ |
| 2 | $1.0 \times 10^{-6}$ | $2.65 \times 10^{2}$ |
| 3 | $1.0 \times 10^{-5}$ | $3.87 \times 10^{2}$ |
| 4 | $1.0 \times 10^{-4}$ | $4.67 \times 10^{2}$ |
| 5 | $1.0 \times 10^{-3}$ | $4.82 \times 10^{2}$ |
| 6 | $1.0 \times 10^{-2}$ | $2.84 \times 10^{2}$ |
| 7 | $1.0 \times 10^{-1}$ | $4.42 \times 10^{2}$ |
| 8 | $1.0 \times 10^{0}$ | $1.32 \times 10^{2}$ |
| 9 | $1.0 \times 10^{1}$ | $3.02 \times 10^{1}$ |
| 10 | $1.0 \times 10^{2}$ | $1.45 \times 10^{1}$ |
| 11 | $1.0 \times 10^{3}$ | $3.02 \times 10^{0}$ |

Table 2: Results – Source: Authors.

| Solicitation | a | b | Iterations |
|---|---|---|---|
| 3.1 | $5.41 \times 10^{-4}$ | $6.33 \times 10^{-1}$ | 13 |
| 3.2 | $5.41 \times 10^{-4}$ | $6.33 \times 10^{-1}$ | 15 |
| 3.3 | $5.41 \times 10^{-4}$ | $6.33 \times 10^{-1}$ | 14 |

## 5. CONCLUSIONS

This paper presents an algorithm to determine the damage characteristic curve (C *vs* S) of asphalt mixtures. Results demonstrate the efficiency of the algorithm, since the same values of the parameters of the damage curve are obtained for the three types of solicitation considered, and these coincide with the original values used in the simulation. However, it has been observed that the values of the parameters $a$ and $b$ may not converge, depending on the value of the $\alpha$, and the intensity of the solicitation. The calculation of the pseudo strains with the use of the theoretical responses in stress for a strain controlled solicitation has the advantage of not needing to obtain the strains in the specimen during the fatigue lab test. This facilitates the testing and subsequent treatment of the data to obtain the C *vs* S curve.

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